Some Problems on Time Change of Gravity Part 2. On Analytical Treatments for Data of the Tidal Variation of Gravity

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By

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Abstract

In case of treating the earth tidal data observed by gravimeter, it is usual that a difference arises in the obtained results depending upon the method of analysis used. In the present article, the effects by applying different methods of drift-elimination and those of harmonic analysis upon the analytical results, are in detail discussed.

Concerning the problem of determination and elimination of instrumental drift, efforts have been made from the standpoint of both accuracy of the results and labour of the calculation. After a detailed study, several methods are derived newly more accurate and excellent compared with any one hitherto adopted. Moreover, a correction factor is introduced newly by the present author for the purpose of eliminating fully the drift.

In order to investigate the methodical comparison of harmonic analysis, different methods of it, Lecolazet's, Doodson-Lennon's and Darwin's, have been applied to the data obtained with the Askania gravimeter No. 111 at eleven stations in Japan. These methods themselves in detail have been compared from the standpoint of accuracy of the analytical results, efficiency of the drift-elimination, labour of the calculation and of a few other points.

1. Introduction

The first successful observation of earth tides with tiltmeter was carried out by E. von Rebeur-Paschwitz in 1892 (1), but that with gravimeter was delayed mainly owing to technical difficulties. In 1914, W. Schweydar (2) made the first observation, with the gravimeter of the bifilar suspension type at Potsdam, for the tidal variation of gravity. Hence for about 50 years, observations of the tidal variation of gravity were largely developed by completion of a highly sensitive gravimeter and in combination with an automatic recording apparatus, and a great many of observations were recently made at various regions of the world. The observations of the tidal variation of gravity made since the beginning of the International Geophysical Year, were especially conclusive in this field in the point of their close cooperation in simultaneous observation at various countries. Some of these observed results were already published making a great contribution to various fields of geophysics.

However the values of tidal factor of gravity and phase lag hitherto obtained by many observations were found to be greatly diverse. It seemed that the diversity was too large to be attributed solely to the observational error. The value of the tidal factor of gravity was found to correlate with rigidity and density distributions of the earth's interior. According to theoretical investigations by H. Takeuchi (**3**), H. Jeffreys (**4**), and C. L. Pekeris and others (**5**), its value was estimated to be about 1.15–1.22, although there were a few differences due to earth-models adopted by them. It was therefore considered that the diversity in the obtained tidal factor was due to disturbances of some causes. Concerning the causes of its diversity, as was already described in the previous article (**6**) of the present study, position of the observation station, effect of the oceanic tides, influence of the local geological structure, effect of the meteorological disturbances, difference of the instrument used in observation, difference of the time of observation, difference of length of the observation period, methodical differences in treating the data, as well as other influences were probably dominant.

Among these various causes, what are regarded exercise exceedingly important effect upon the tidal factor of gravity, are of the oceanic tides, the geological structure and the meteorological disturbances. Concerning these effects, some were already investigated in the previous article (**6**), and others in the next article (**7**). On the contrary, the effects caused by differences in the instrument, the time of observation and the method of treating the data, are regarded not so prominent in comparison with those of the oceanic tides, etc. But when observational results obtained at various stations in the world are compared, due considerations must be paid to each of them. The first point, that is, to what extent the difference in instrument affects the obtained results, can be investigated by a simultaneous observation with more than two gravimeters ; the second point, by long continuous observations ; and the third point of comparison, by theoretical consideration or by comparison with the analytical results obtained by practically applying various methods to the observed data.

When the sensitivity of instrument was low, the observational error was considerably large, when the diversity of results introduced by methodical difference in treating the data was smaller than the observational error, it was then unnecessary to take account of it. By the completion of highly sensitive gravimeter with automatic recording apparatus and the recent development of measuring technique, however, the accuracy of observation was largely increased and consequently the observational error decreased. Consequently, in case of recent observation, the methodical difference of treating the data should be considered in discussing observational results. Under such circumstances, a resolution was made at the Third International Symposium on Earth Tides held at Trieste in 1959 (8) pertaining to the method of analysis. In the following these problems are in detail discussed.

Now, the treatment of the earth tidal data observed by gravimeter, equipped with automatic recording apparatus, is generally divided into three processes of analysis. The first is the reading of hourly values from the registrograms ; the second, elimination of instrumental drift ; and the third, the harmonic analysis.

The first process is also composed of the reading of the original registrogram, the correction for disturbance and jump on the recording, and complement for the part of interruption in the recording. Concerning the reading of the registrogram, as already described in the previous article (**6**) of the present study, results have been reported by some investigators. Concerning the effect upon analytical results of misreading of the registrogram, there is a research by N. N. Pariisky and others (**9**), while concerning that by disturbance and jump on the recording, there are researches by M. S. Reford (**10**) and others (**9**). After all, their emphasis is that such disturbances and jumps on the recording must be entirely excluded. In practical observation, interruption in recording cannot be avoidable. Its part must be interpolated by some suitable methods. Such interpolations have recently been discussed by I. M. Longman (**11**, **12**), A. P. Venedikov (**13**) and R. Lecolazet (**14**). There is no problem when the observation is made by zero-method, but when the change in inclination of mass lever arm of the gravimeter, corresponding to gravity change, is magnified and recorded on the recording paper, correction for the variation of electric voltage must be made in the first process, if necessary. Furthermore, when the scale of galvanometer for recording is not linear, the correction for non-linearity must be made for the values of reading. By such the first process, a series of successive hourly values are obtained, and then the subsequent processes will be proceeded with using them.

In the following the detailed considerations concerning the elimination of drift and harmonic analysis are discussed.

2. Determination and elimination of drift

In case of treating the earth tidal data observed by gravimeter, it is a difficult but important problem how to eliminate fully the drift. Study concerning this problem has been made by many investigators. As the method available, a successive moving mean of 25 hours-reading has been used customarily for a long time.

But in 1941, A. T. Doodson and H. D. Warburg (**15**) devised an excellent one, 30 hours' selected mean, as a method for estimation of a mean sea level. It was usually called "Admiralty method" and could also be applied for determining the drift curve of gravimeter. This method was ascertained to be excellent by many investigators (**10**, **16**).

Generally speaking, it is necessary that determination of the drift curve of gravimeter, or in other words, determination of the zero line, must be viewed from two standpoints. Namely, one is to determine the drift as precisely as possible and the other to save the labour of calculation as much as possible. The Admiralty method is satisfactory to the former, but not to the latter.

The efficient method for these two points was devised by B. P. Pertzev in 1957 (**17**). His method is a 15 hours' selected mean and has succeeded in simplifying the Admiralty method without any lowering in accuracy. The Pertzev's method was ascertained to be excellent at the Third International Symposium on Earth Tides above-mentioned, and determined as the standard for eliminating the drift from the data of earth tides observed by gravimeter. The relation between both methods of Admiralty and Pertzev was in detail discussed by P. Melchior (**18**) and G. W. Lennon (**19**). It was also ascertained by V. G. Balenko (**20**) that the Pertzev's method was the perfect. Besides the above a study concerning the method of determining the drift was also made by K. Lassovszky (**21**), A. Gougenheim (**22**), K. Rinner (**23**) and others.

The persons above-mentioned were those who had opinion that the drift must first of all be determined and eliminated from all the observed values prior to a harmonic analysis. But there were those who took a different standpoint. R. Lecolazet (**24**), A. T. Doodson and G. W. Lennon (**25**), C. T. Suthons (**26**) and others tried to eliminate the drift in the process of calculation of harmonic analysis. Speaking in detail, R. Lecolazet (**24**) devised his own method of harmonic analysis in 1956. In this method, he described to eliminate automatically the drift in the process on analysis under assumption that the drift curve was represented by a polynomial of second power with time. A. T. Doodson and G. W. Lennon (**25**) devised a method how the drift was eliminated by applying a simple linear transformation to hourly observed values, and they obtained good results by combining it with their own method of harmonic analysis. C. T. Suthons (**26**) had also devised one in which the drift was graphically eliminated.

In the following, a detailed consideration concerning determination and elimination of drift is described summarily from the standpoint of both accuracy of results and labour of calculation, taking the standpoint that the drift must be eliminated from the observed values before commencing an analysis.

(1) Theoretical consideration

Let Y_t be the observed value at some hour t, Y_t is given by the following formula,

$$Y_t=\sum R_n \cos(\omega_n t+\varepsilon_n)+D(t), \tag{2.1}$$

where ω_n : the speed of a constituent 'n' in degrees per mean solar hour,

R_n : the amplitude of the constituent 'n' with speed of ω_n,

ε_n : the phase of the constituent 'n' at the origin of time,

and $D(t)$: the term due to the drift.

To discuss the present problem, next three conditions are adopted :

(i) Observed successive hourly values are used in calculation.

(ii) The time range of values which is used in calculation is symmetrical about hour t_0 and is limited within 24 hours.

(iii) The labour of calculation should be reduced as much as possible.

Summing up the equation (2.1) (let m be the number of summation) and dividing by m,

$$\frac{1}{m}\Sigma Y_t=\frac{1}{m}\Sigma\Sigma R_n\cos(\omega_n t+\varepsilon_n)+\frac{1}{m}\Sigma D(t)\,. \tag{2.2}$$

Transforming the first term on the right-hand side in (2.2) to a product form, and expanding the second one in a Taylor series around $t=t_0$, then the equation (2.2) leads to

$$\frac{1}{m}\Sigma Y_t=\Sigma A_n R_n\cos(\omega_n t_0+\varepsilon_n)+D(t_0)+\alpha D''(t_0)+\beta D^{\mathrm{IV}}(t_0)+\cdots\cdots, \tag{2.3}$$

where $\alpha=\frac{1}{m}\Sigma\frac{h^2}{2}$, $\beta=\frac{1}{m}\Sigma\frac{h^4}{4!}$, and h is expressed by a half of the time range of calculation as its unit. In (2.3), the values of the left-hand side alone are determined by observation. In order to obtain the drift $D(t_0)$ at the time of $t=t_0$, therefore, it needs to make a suitable combination of t so as to be zero or very small of the coefficient A_n of the first term on the right-hand side of (2.3) for the diurnal and semi-diurnal constituents.

The fundamental forms are firstly considered. The coefficients A_n in cases of $m=2$, 3 and 5 are given in Table 2.1.

Table 2.1.

	m	t	Coefficient A_n	Condition
(I)	2	$t_0\pm p$	$\cos p\omega_n$	$0<p\leqq 12$, integer
(II)	3	t_0, $t_0\pm p$	$\frac{1}{3}\cdot\frac{\sin 1.5p\omega_n}{\sin 0.5p\omega_n}$	$0<p\leqq 12$, integer
(III)	5	t_0, $t_0\pm p$, $t_0\pm q$	$\frac{1}{5}\left(1+4\cos\frac{p+q}{2}\omega_n\cdot\cos\frac{p-q}{2}\omega_n\right)$	$0<p<q\leqq 12$, integers

Relations between the coefficient A_n and the speed ω_n in cases of (I), (II) and (III) are shown in Fig. 2.1. In this figure, several parts of p or p and q which fulfil the conditions given in Table 2.1 are shown alone.

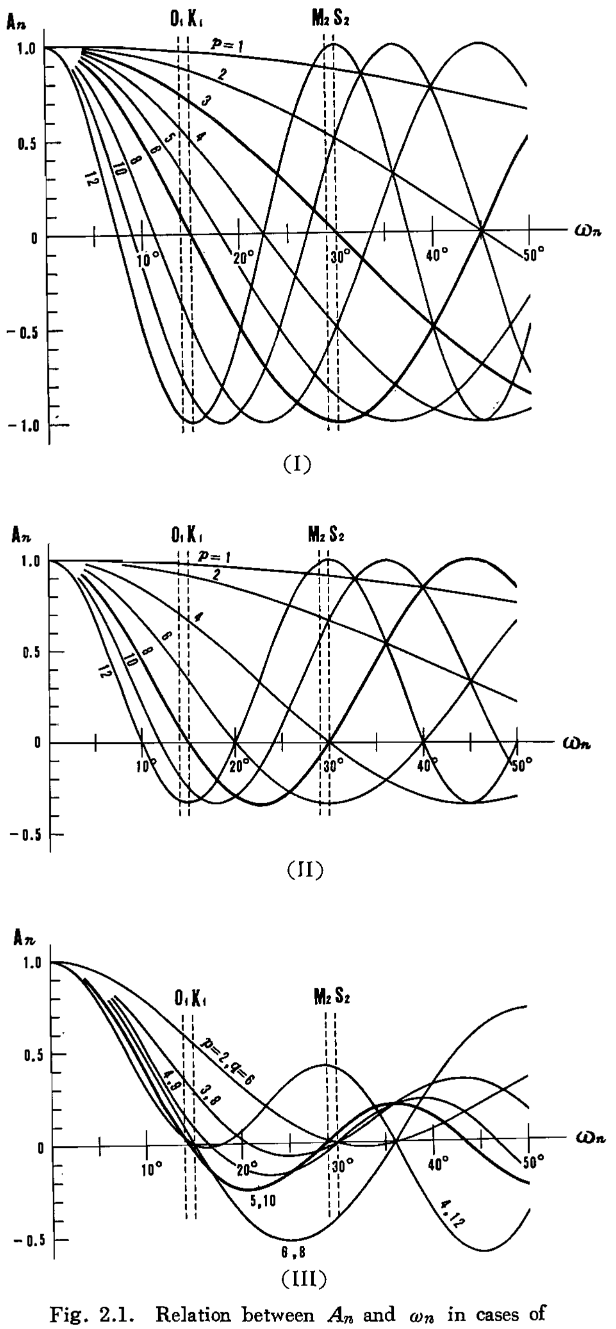

Fig. 2.1. Relation between A_n and ω_n in cases of (I) m=2, (II) m=3 and (III) m=5.

Table 2.2.

Method	m	t	Coefficient A_n
(a)	2	$t_0 \pm 1$	$\cos \omega_n$
(b)	2	$t_0 \pm 3$	$\cos 3\omega_n$
(c)	2	$t_0 \pm 6$	$\cos 6\omega_n$
(d)	3	$t_0,\ t_0 \pm 8$	$\frac{1}{3} \cdot \frac{\sin 12\omega_n}{\sin 4\omega_n}$
(e)	5	$t_0,\ t_0 \pm 5,\ t_0 \pm 10$	$\frac{1}{5} \cdot \frac{\sin 12.5\omega_n}{\sin 2.5\omega_n}$

Table 2.3.

Method	m	t	Coefficient A_n
(bc)	4	3,9	$\frac{1}{4} \cdot \frac{\sin 12\omega_n}{\sin 3\omega_n}$
(ad)	6	1,7,9	$\frac{1}{3} \cdot \frac{\sin 12\omega_n}{\sin 4\omega_n} \cdot \cos \omega_n$
(bd)	6	3,5,11	$\frac{1}{3} \cdot \frac{\sin 12\omega_n}{\sin 4\omega_n} \cdot \cos 3\omega_n$
(cd)	6	2,6,14	$\frac{1}{3} \cdot \frac{\sin 12\omega_n}{\sin 4\omega_n} \cdot \cos 6\omega_n$
(ae)	10	1,4,6,9,11	$\frac{1}{5} \cdot \frac{\sin 12.5\omega_n}{\sin 2.5\omega_n} \cdot \cos \omega_n$
(be)	10	2,3,7,8,13	$\frac{1}{5} \cdot \frac{\sin 12.5\omega_n}{\sin 2.5\omega_n} \cdot \cos 3\omega_n$
(ce)	10	1,4,6,11,16	$\frac{1}{5} \cdot \frac{\sin 12.5\omega_n}{\sin 2.5\omega_n} \cdot \cos 6\omega_n$
(bcd)	12	1,3,5,9,11,17	$\frac{1}{12} \cdot \frac{\sin 12\omega_n}{\sin 3\omega_n} \cdot \frac{\sin 12\omega_n}{\sin 4\omega_n}$
(de)	15	0,2,3,5,8,10,13,18	$\frac{1}{15} \cdot \frac{\sin 12\omega_n}{\sin 4\omega_n} \cdot \frac{\sin 12.5\omega_n}{\sin 2.5\omega_n}$
(ade)	30	$\underline{1}$,2,3,$\underline{4}$,6,7,$\underline{9}$,11,12,14,17,19	$\frac{1}{15} \cdot \frac{\sin 12\omega_n}{\sin 4\omega_n} \cdot \frac{\sin 12.5\omega_n}{\sin 2.5\omega_n} \cdot \cos \omega_n$
(bde)	30	$\underline{0}$,1,2,3,$\underline{5}$,6,7,8,10,11,13,15,16,21	$\frac{1}{15} \cdot \frac{\sin 12\omega_n}{\sin 4\omega_n} \cdot \frac{\sin 12.5\omega_n}{\sin 2.5\omega_n} \cdot \cos 3\omega_n$
(cde)	30	1,2,3,$\underline{4}$,6,7,8,9,11,12,14,16,19,24	$\frac{1}{15} \cdot \frac{\sin 12\omega_n}{\sin 4\omega_n} \cdot \frac{\sin 12.5\omega_n}{\sin 2.5\omega_n} \cdot \cos 6\omega_n$
(24)	24	Successive moving mean of 24 hours	$\frac{1}{24} \cdot \frac{\sin 12\omega_n}{\sin 0.5\omega_n}$
(25)	25	Successive moving mean of 25 hours	$\frac{1}{25} \cdot \frac{\sin 12.5\omega_n}{\sin 0.5\omega_n}$

Notes The figures on t-column show p in $t_0 \pm p$ (see Table 2.2).
The values of hour with underline be treated as weight by 2 and the others as weight by 1.

Table 2.4. Values of the coefficients A_n, α and β

Method	A_n								α	β
	M_2	S_2	N_2	K_2	K_1	O_1	P_1	Q_1		
(a)	0.87476	0.86603	0.87937	0.86530	0.96578	0.97051	0.96603	0.97278	0.50000	0.04167
(b)	0.05321	——	0.08165	−0.00436	0.70567	0.74509	0.70855	0.76380	0.50000	0.04167
(c)	−0.99434	−1.00000	−0.98667	−0.99996	−0.00436	0.11060	0.00436	0.16677	0.50000	0.04167
(d)	−0.07822	——	−0.11688	0.00663	−0.00335	0.08852	0.00336	0.13631	0.33333	0.02778
* (e)	0.00842	0.05359	−0.01659	0.05714	−0.04571	0.03486	−0.04003	0.07853	0.25000	0.01771
(bc)	−0.05291	——	−0.08056	0.00436	−0.00308	0.08241	0.00309	0.12738	0.27778	0.02109
(ad)	−0.06843	——	−0.10278	0.00574	−0.00324	0.08591	0.00325	0.13260	0.26955	0.01897
* (bd)	−0.00416	——	−0.00954	−0.00003	−0.00237	0.06596	0.00238	0.10411	0.21350	0.01456
(cd)	0.07778	——	0.11532	−0.00663	0.00001	0.00979	0.00001	0.02273	0.20068	0.01436
(ae)	0.00736	0.04641	−0.01459	0.04944	−0.04415	0.03383	−0.03868	0.07639	0.21074	0.01295
* (be)	0.00045	——	−0.00135	−0.00025	−0.03226	0.02598	−0.02837	0.05998	0.17456	0.01026
(ce)	−0.00837	−0.05359	0.01636	−0.05713	0.00020	0.00386	−0.00017	0.01310	0.16797	0.01039
*(bcd)	0.00414	——	0.00942	0.00003	0.00001	0.00729	0.00001	0.01736	0.15167	0.00877
* (de)	−0.00066	——	0.00194	0.00038	0.00015	0.00309	−0.00013	0.01070	0.14300	0.00785
*(ade)	−0.00058	——	0.00170	0.00033	0.00015	0.00300	−0.00013	0.01041	0.12973	0.00650
*(bde)	−0.00004	——	0.00016	−0.00000	0.00011	0.00230	−0.00010	0.00818	0.11527	0.00533
*(cde)	0.00065	——	−0.00191	−0.00038	−0.00000	0.00034	−0.00000	0.00179	0.11169	0.00516
(24)	−0.03512	——	−0.05444	0.00276	−0.00278	0.07543	0.00279	0.11755	0.18116	0.00982
* (25)	0.00641	0.04000	−0.01278	0.04259	−0.04265	0.03285	−0.03734	0.07430	0.18056	0.00976

Notes Among the figures shown in this table, —— shows the perfect zero, while 0.00000 denotes that the significant figure appears below the sixth place after decimal. The methods with asterisk are those ascertained to be excellent.

In case (I), there exists no value of p to make zero the coefficient A_n for both diurnal and semi-diurnal constituents. Then, the following three cases are adopted :

(a) $p=1$,

(b) $p=3$ (The value of A_n is nearly equal to zero for all semi-diurnal constituents), and

(c) $p=6$ (The value of A_n is nearly equal to zero for all diurnal constituents).

In case (II), only when

(d) $p=8$,

within the range of above-mentioned condition, the values of the coefficient A_n for both constituents become almost zero. Moreover, in case (III),

(e) $p=5$, $q=10$

serve the present purpose. Coefficients A_n for these five cases are shown in Table 2.2.

A. T. Doodson and H. D. Warburg (**15**) have pointed out three special groupings of above-described (a), (d) and (e).

Table 2.3 shows the values of t and A_n of various combinations based on these five methods. The methods given in Table 2.3 are the ones to fulfil the above-mentioned conditions and to bring the value of A_n within 0.1 for all the principal constituents, among selected means obtained by

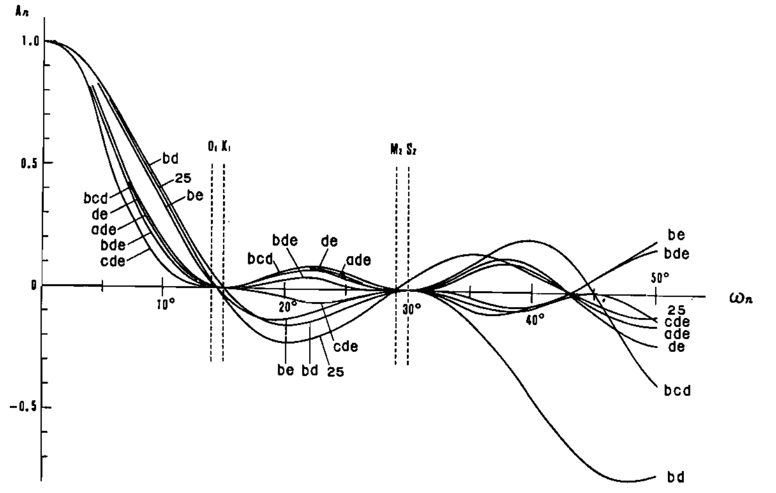

Fig. 2.2. Relation between A_n and ω_n.

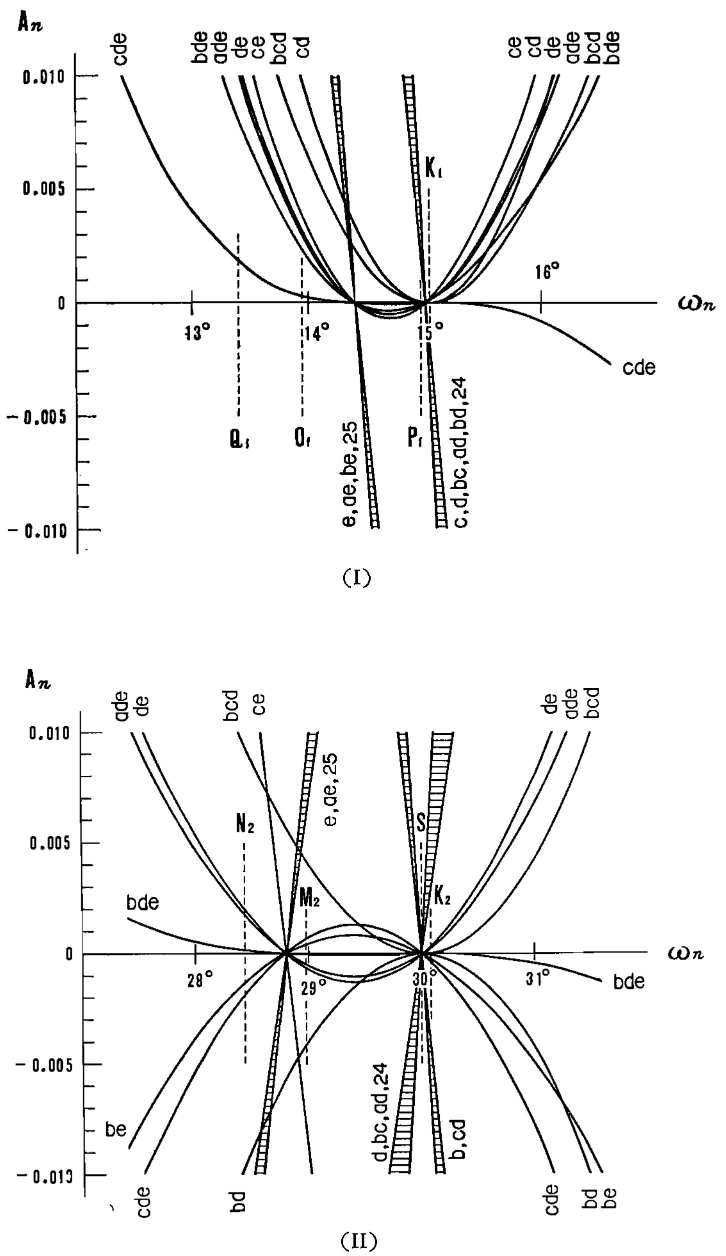

Fig. 2.3. Relation between A_n and ω_n at the neighbourhood of (I) the diurnal and (II) semi-diurnal constituents.

combining of the fundamental methods. In Table 2.3, moving means of successive 24 and 25 hours used customarily are contained for comparison purposes. In this table, the method (de) is one established by B. P. Pertzev (**17**), and the method (ade) by A. T. Doodson and H. D. Warburg (**15**).

Relations between the coefficient A_n and the speed ω_n for some methods in Table 2.3 are shown in Fig. 2.2. Fig. 2.3 is made up of magnifying the neighbourhood of the diurnal and semi-diurnal constituents in Fig. 2.2.

Values of A_n for the principal constituents of the diurnal and semi-diurnal tides and those of the coefficients α and β of derivatives on the right-hand side in (2.3) are shown in Table 2.4.

As can easily be understood from the Table 2.4, Figs. 2.2 and 2.3, the moving mean of 25 hours is satisfactory to the lunar constituents, but not to the solar. On the contrary, the moving mean of 24 hours is satisfactory to the solar constituents, but not to the lunar. Pertzev's method (de) is very accurate notwithstanding that it has a small value of m in comparison with the moving mean. Moreover, in Pertzev's method (de), the labour in calculation is halved in comparison with the Admiralty method (ade), although both accuracies in the determination of drift are almost equivalent.

But, in order to obtain the form of drift curve itself in detail, a more accurate method than Pertzev's is required. Especially, when the electronic computing machine is available in calculation as in the recent times, the condition (iii), that is, the reduction of labour is needless and consequently the more accurate method to be applied is required emphatically even if its calculation becomes somewhat troublesome. Methods to meet this requirement are (bde) and (cde). The methods (bde) and (cde) are quite the same as the Admiralty one (ade) in labour, and the former is a better method for the semi-diurnal constituents, while the latter for the diurnal ones.

Nevertheless, when reduction of labour in calculation is persistently required, the method (be) or (bcd) is available. Both these methods are far excellent ones compared with the moving mean of 25 hours. In extreme case, the accuracy with the same degree as the moving mean of 25 hours can be obtained even by applying the method (e).

From these considerations, the effective methods to determine the drift are nine of them marked with asterisk in Table 2.4. In practical calculation, the iterative scheme with help of two or three stencils is convenient.

It saves much labour in calculation and controls it.

The above-mentioned were mainly concerning the coefficient A_n alone and no consideration was given to the form of the drift curve. Needless to say, if a more accuracy is required for determination of the drift, further thought is required for its form. Generally speaking, however, since the drift usually changes slowly, the second derivative of the drift curve is small. Then, one can simply put

$$D(t_0)=\frac{1}{m}\sum Y_t, \tag{2.4}$$

according to each method. But when the drift curve has a large curvature, in other words, in cases of sudden changes of temperature and atmospheric pressure, the second derivative should be taken into account. In these cases, as can be understood from Table 2.4, since its coefficient is small, the second derivative can be evaluated to some degree by assuming it as equal to the second difference, as proposed by M. S. Reford (**10**), or by approximating the drift curve to a parabola, as by B. P. Pertzev (**17**). In fact, M. S. Reford (**10**) has investigated the effect of correction for the curvature of drift upon results of the harmonic analysis, based on the data obtained by North American gravimeter. According to his results, it is concluded that the results could differ for the main constituents by only 0.1% in amplitude and 0.3° in phase lag. As to this problem, the author is now investigating it and will report in near opportunity.

The methods (bde) and (cde) are excellent for the present purpose. When the method (bde) or (cde) is applied, however, the value of the coefficient A_n is not perfect zero for all the main constituents but has a certain definite value. This indicates that some little portions of constituents are subtracted and others added excessively with the drift in the calculation process of drift-elimination.

Then, at the hour $t=t_0$, the equation (2.1) gives

$$Y_{t0}=\sum R_n\cos(\omega_n t_0+\varepsilon_n)+D(t_0). \tag{2.5}$$

Assuming the terms lower than the second derivative of the drift curve to be negligible, the equation (2.3) leads to

$$\frac{1}{m}\sum Y_t=\sum A_n R_n\cos(\omega_n t_0+\varepsilon_n)+D(t_0). \tag{2.6}$$

Eliminating the drift term $D(t_0)$ from the above two equations (2.5) and

(2.6),

$$Y_{t0}-\frac{1}{m}\Sigma Y_t=\Sigma(1-A_n)R_n\cos(\omega_n t_0+\varepsilon_n). \tag{2.7}$$

The left-hand side of the equation (2.7) is the data after the application of reduction for the drift. Therefore, the equation (2.7) indicates that the amplitudes of harmonic constituents after elimination of the drift by the method given in Tables 2.2 and 2.3 are not the real amplitudes but $(1-A_n)$ times as great as them. To obtain the real amplitude, the amplitude obtained from the harmonic analysis must be multiplied by correction factor of $1/(1-A_n)$ proposed by the author (**27**). The values of correction factor for the various methods are given in Table 2.5.

Table 2.5. Values of correction factor $1/(1-A_n)$ for the various methods

Method	M_2	S_2	N_2	K_2	K_1	O_1	P_1	Q_1
(a)	7.98467	7.46436	8.28981	7.42390	29.22268	33.90980	29.48113	36.73769
(b)	1.05620	1.00000	1.08891	0.99566	3.39755	3.92295	3.43112	4.23370
(c)	0.50142	0.50000	0.50335	0.50001	0.99566	1.12435	1.00438	1.20015
(d)	0.92745	1.00000	0.89535	1.00667	0.99666	1.09712	1.00337	1.15782
(e)	1.00849	1.05662	0.98368	1.06060	0.95629	1.03612	0.96151	1.08522
(bc)	0.94975	1.00000	0.92545	1.00438	0.99693	1.08981	1.00310	1.14597
(ad)	0.93595	1.00000	0.90680	1.00577	0.99677	1.09398	1.00326	1.15287
(bd)	0.99586	1.00000	0.99055	0.99997	0.99764	1.07062	1.00239	1.11621
(cd)	1.08434	1.00000	1.13035	0.99341	1.00001	1.00989	1.00001	1.02326
(ae)	1.00741	1.04867	0.98562	1.05201	0.95772	1.03501	0.96276	1.08271
(be)	1.00045	1.00000	0.99865	0.99975	0.96875	1.02667	0.97241	1.06381
(ce)	0.99170	0.94914	1.01663	0.94596	1.00020	1.00387	0.99983	1.01327
(bcd)	1.00416	1.00000	1.00951	1.00003	1.00001	1.00734	1.00001	1.01767
(de)	0.99934	1.00000	1.00194	1.00038	1.00015	1.00310	0.99987	1.01082
(ade)	0.99942	1.00000	1.00170	1.00033	1.00015	1.00301	0.99987	1.01052
(bde)	0.99996	1.00000	1.00016	1.00000	1.00011	1.00231	0.99990	1.00825
(cde)	1.00065	1.00000	0.99809	0.99962	1.00000	1.00034	1.00000	1.00179
(24)	0.96607	1.00000	0.94837	1.00277	0.99723	1.08158	1.00280	1.13321
(25)	1.00645	1.04167	0.98738	1.04448	0.95909	1.03397	0.96400	1.08026

The methods given in Table 2.3 are good ones within the range of fulfilling the above-mentioned conditions. When the condition (iii) and part of one (ii) are neglected, in other words, the accuracy which determines the drift must be high and no consideration is to be given for labour

Table 2.6.

Method	m	t	Coefficient A_n
(dd)	9	0_3, 8_2, 16_1	$\frac{1}{9}\cdot\left(\frac{\sin 12\omega_n}{\sin 4\omega_n}\right)^2$
(bbcc)	16	0_4, 6_3, 12_2, 18_1	$\frac{1}{16}\cdot\left(\frac{\sin 12\omega_n}{\sin 3\omega_n}\right)^2$
(ee)	25	0_5, 5_4, 10_3, 15_2, 20_1	$\frac{1}{25}\cdot\left(\frac{\sin 12.5\omega_n}{\sin 2.5\omega_n}\right)^2$
(dde)	45	0_3, 2_2, 3_2, 5_3, 6_1, 8_2, 10_3, 11_1, 13_2, 16_1, 18_2, 21_1, 26_1	$\frac{1}{45}\cdot\left(\frac{\sin 12\omega_n}{\sin 4\omega_n}\right)^2\cdot\frac{\sin 12.5\omega_n}{\sin 2.5\omega_n}$
(bcde)	60	0_2, 1_3, 2_1, 3_1, 4_2, 5_2, 6_2, 7_2, 8_1, 9_2, 10_1, 11_2, 12_1, 13_1, 14_1, 15_1, 16_1, 17_1, 19_1, 21_1, 22_1, 27_1	$\frac{1}{60}\cdot\frac{\sin 12\omega_n}{\sin 3\omega_n}\cdot\frac{\sin 12\omega_n}{\sin 4\omega_n}\cdot\frac{\sin 12.5\omega_n}{\sin 2.5\omega_n}$

Values of the coefficients A_n, α and β

Method	A_n								α	β
	M_2	S_2	N_2	K_2	K_1	O_1	P_1	Q_1		
(dd)	0.00612	——	0.01366	0.00004	0.00001	0.00784	0.00001	0.01858	0.16667	0.01042
(bbcc)	0.00280	——	0.00649	0.00002	0.00001	0.00679	0.00001	0.01623	0.13889	0.00746
(ee)	0.00007	0.00287	0.00028	0.00326	0.00209	0.00122	0.00160	0.00617	0.12500	0.00612
(dde)	0.00005	——	−0.00023	0.00000	−0.00000	0.00027	−0.00000	0.00146	0.10010	0.00422
(bcde)	0.00003	——	−0.00016	0.00000	−0.00000	0.00025	−0.00000	0.00136	0.09442	0.00377

Values of correction factor $1/(1-A_n)$ for the main constituents

Method	M_2	S_2	N_2	K_2	K_1	O_1	P_1	Q_1
(dd)	1.00616	1.00000	1.01385	1.00004	1.00001	1.00790	1.00001	1.01893
(bbcc)	1.00281	1.00000	1.00653	1.00002	1.00001	1.00684	1.00001	1.01650
(ee)	1.00007	1.00288	1.00028	1.00327	1.00209	1.00122	1.00160	1.00621
(dde)	1.00005	1.00000	0.99977	1.00000	1.00000	1.00027	1.00000	1.00146
(bcde)	1.00003	1.00000	0.99984	1.00000	1.00000	1.00025	1.00000	1.00136

Note : The suffix on t-column in the upper table shows the weight.

Table 2.7. Results obtained from the harmonic

Observation station :
Sensitivity of instrument :
Harmonic analysis :
Central epoch :

Method	M_2		S_2		N_2	
	G	κ	G	κ	G	κ
(O)	1.143±0.003	−0.°13±0.°13	1.081±0.005	−4.°87±0.°28	1.202±0.013	−10.°26±0.°61
(d)	1.230±0.003	0.00±0.13	1.073±0.006	−5.24±0.29	1.336±0.013	−11.43±0.58
(e)	1.132±0.003	−0.10±0.14	1.014±0.006	−5.35±0.31	1.213±0.014	−11.50±0.64
(bd)	1.149±0.004	−0.15±0.16	1.076±0.007	−5.05±0.33	1.219±0.015	−10.91±0.71
(be)	1.142±0.003	−0.10±0.14	1.082±0.006	−5.10±0.28	1.206±0.012	−11.03±0.59
(bcd)	1.135±0.004	−0.20±0.16	1.079±0.006	−4.82±0.33	1.188±0.015	− 9.75±0.69
(de)	1.141±0.003	−0.23±0.14	1.079±0.005	−4.99±0.28	1.200±0.013	− 9.91±0.63
(ade)	1.141±0.003	−0.15±0.13	1.081±0.005	−5.12±0.26	1.202±0.012	− 9.53±0.58
(bde)	1.142±0.003	−0.15±0.14	1.080±0.006	−5.00±0.28	1.202±0.012	−10.43±0.59
(cde)	1.142±0.003	−0.08±0.14	1.080±0.005	−5.04±0.26	1.204±0.012	−10.78±0.56
(24)	1.181±0.003	−0.45±0.13	1.078±0.005	−5.32±0.28	1.261±0.012	−11.88±0.56
(25)	1.133±0.003	+0.25±0.14	1.036±0.005	−5.94±0.28	1.202±0.013	−12.11±0.61

Observation station :
Sensitivity of instrument :
Harmonic analysis :
Central epoch :

Method	M_2		S_2		N_2	
	G	κ	G	κ	G	κ
(O)	1.114±0.004	−1.°24±0.°19	1.205±0.010	−1.°29±0.°51	1.216±0.022	−0.°82±1.°06
(d)	1.207±0.004	−1.42±0.18	1.213±0.010	−1.30±0.49	1.343±0.022	−1.29±0.93
(e)	1.107±0.004	−1.44±0.18	1.134±0.011	−2.19±0.56	1.227±0.023	−1.47±1.08
(bd)	1.123±0.005	−1.20±0.21	1.206±0.012	−2.17±0.59	1.212±0.027	−0.96±1.26
(be)	1.116±0.004	−1.17±0.18	1.216±0.010	−0.90±0.49	1.219±0.021	−0.86±0.99
(bcd)	1.108±0.004	−1.34±0.21	1.200±0.011	−1.20±0.58	1.202±0.025	−1.54±1.14
(de)	1.115±0.004	−1.24±0.19	1.204±0.010	−1.32±0.51	1.204±0.023	−1.01±1.08
(ade)	1.115±0.004	−1.19±0.18	1.206±0.009	−1.24±0.46	1.212±0.020	−1.41±0.94
(bde)	1.114±0.004	−1.19±0.18	1.203±0.009	−1.55±0.48	1.201±0.021	−2.42±1.01
(cde)	1.115±0.004	−1.22±0.16	1.206±0.009	−1.29±0.43	1.221±0.019	−1.04±0.83
(24)	1.155±0.004	−1.72±0.18	1.209±0.010	−1.39±0.49	1.282±0.022	−1.94±0.98
(25)	1.109±0.004	−1.32±0.18	1.164±0.010	−1.47±0.53	1.229±0.022	−0.94±1.06

Note: The method (O) is a result obtained from the data containing the drift, and the others analysis after elimination of drift by various methods

Shionomisaki
2.5141 μgal/mm
Lecolazet's method
Feb. 3, 18h, 1958 (UT)

K_1		O_1		Q_1	
G	κ	G	κ	G	κ
1.096±0.004	−8°.75±0°.21	1.098±0.006	+1°.30±0°.33	0.708±0.026	+ 4°.43±2°.18
1.126±0.004	−7.98±0.23	0.955±0.007	+0.81±0.39	0.708±0.028	+12.13±2.36
1.179±0.005	−8.30±0.21	1.023±0.007	+1.16±0.38	0.764±0.028	+16.69±2.18
1.128±0.005	−8.47±0.24	0.984±0.008	+0.85±0.44	0.754±0.032	+ 6.93±2.46
1.156±0.004	−8.32±0.19	1.029±0.007	+0.45±0.36	0.778±0.026	+ 7.44±1.94
1.123±0.005	−8.65±0.23	1.070±0.008	+0.60±0.39	0.797±0.030	+ 9.41±2.23
1.121±0.005	−8.66±0.19	1.074±0.007	+0.46±0.34	0.796±0.027	+ 8.83±1.98
1.121±0.004	−8.58±0.21	1.070±0.006	+0.53±0.33	0.815±0.025	+ 8.49±1.79
1.122±0.004	−8.77±0.21	1.067±0.006	+0.21±0.33	0.812±0.025	+ 7.03±1.83
1.120±0.004	−8.60±0.19	1.074±0.006	+0.76±0.31	0.818±0.025	+ 8.91±1.81
1.128±0.004	−8.07±0.19	0.976±0.007	+1.18±0.36	0.732±0.026	+12.46±2.09
1.172±0.004	−8.25±0.19	1.019±0.007	+0.81±0.34	0.760±0.026	+10.78±2.01

Kyoto (II)
2.5248 μgal/mm
Lecolazet's method
June 27, 18h, 1958 (UT)

K_1		O_1		Q_1	
G	κ	G	κ	G	κ
1.038±0.005	−1°.30±0°.24	1.017±0.008	−1°.09±0°.44	1.204±0.046	+14°.65±2°.49
1.041±0.005	−1.46±0.23	0.923±0.008	−0.83±0.48	0.993±0.045	+15.65±2.96
1.083±0.005	−1.03±0.23	0.974±0.008	−1.46±0.48	1.099±0.047	+16.20±2.79
1.037±0.005	−1.60±0.28	0.953±0.009	−1.78±0.56	1.086±0.054	+16.79±3.26
1.072±0.004	−1.58±0.21	0.993±0.008	−0.81±0.43	1.038±0.044	+10.29±2.96
1.036±0.005	−1.23±0.24	1.010±0.009	−1.34±0.49	1.116±0.050	+13.30±3.06
1.037±0.005	−1.41±0.23	1.011±0.008	−1.28±0.44	1.138±0.046	+13.17±2.73
1.037±0.004	−1.35±0.21	1.012±0.007	−1.41±0.39	1.135±0.041	+12.92±2.36
1.039±0.004	−1.28±0.23	1.008±0.008	−1.41±0.41	1.134±0.043	+14.42±2.48
1.038±0.004	−1.38±0.19	1.016±0.007	−1.36±0.38	1.149±0.039	+14.00±2.18
1.040±0.004	−1.33±0.23	0.940±0.008	−0.71±0.46	1.008±0.044	+15.34±2.89
1.083±0.005	−1.31±0.23	0.984±0.008	−1.34±0.46	1.040±0.046	+14.07±2.96

being results obtained from the data after elimination of the drift by respective method.

saving, the desirable method to meet the requirement can immediately be derived. Methods (dde) and (bcde) shown in Table 2.6 are examples. The effective methods (dd), (bbcc) and (ee) to determine the drift among weighted means, obtained by combining the fundamental methods shown in Table 2.2, are also shown in Table 2.6. In fact, the method (bcde) with $m=60$ is the most accurate and excellent one in view of obtaining in detail the form of the drift curve.

(2) Application to the observational data

In Table 2.7 are shown results of a harmonic analysis by Lecolazet's method (28) after elimination of the drift by each method mentioned above. The data observed at two stations—Shionomisaki and Kyoto (II)—are used as examples. They are obtained by means of the Askania Gs-11 gravimeter No. 111 with automatic recording apparatus. The reason why the data observed at Shionomisaki and Kyoto (II) have been selected as examples in the present consideration is that, among one month's observations of the tidal variation of gravity at eleven stations in Japan, the drift curve at Kyoto (II) being good in its linearity, while the change of drift at Shionomisaki irregular and its deviation large. Namely, two extreme cases are treated. In Table 2.7 are also shown analytical results obtained by applying Lecolazet's method (28) directly to the data containing the drift for the purpose of comparison.

The drift curves obtained by various methods are shown in Fig. 2.4 with graphs of room temperature and atmospheric pressure in the corresponding period.

Effect upon the tidal factor of gravity and the phase lag caused by difference in the method of eliminating the drift is clearly seen in the Table 2.7. Even for the most reliable M_2-constituent among the ones obtained from the data of one month, there are differences by about 2% in tidal factor of gravity and by about 0.5° in phase lag. In Table 2.7 are contained the results after elimination of the drift by method (d), the simplest combination to make the coefficient A_n small for both diurnal and semi-diurnal constituents, and those by the moving mean of 24 hours, which has been used customarily, for the purpose of comparison. There are large differences, in methods (d) and (24), compared with others.

As could easily be seen from the Table 2.7, the tendencies of effect

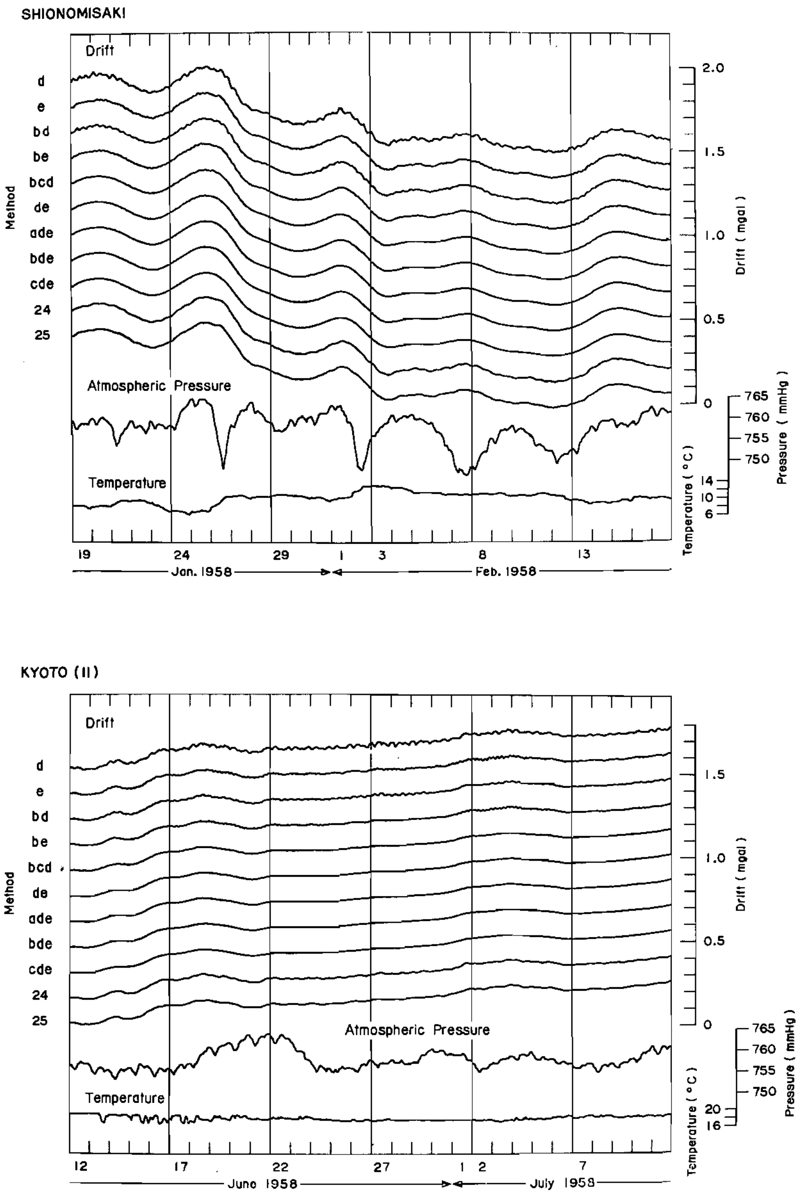

Fig. 2.4. Drift curves obtained by the various methods from the data observed at two stations—Shionomisaki (upper figure) and Kyoto (II) (lower figure) —and the changes of atmospheric pressure and room temperature.

upon the results caused by the method of drift-elimination were quite alike at both stations. This indicated that the form of the drift curve did not exercise considerable effect upon the analytical results, but the method to eliminate itself showed an important influence. This fact was certainly, to some extent, related to the point that the Lecolazet's method was applied to a harmonic analysis. Concerning the principal semi-diurnal constituents, good results were obtained by applying the Lecolazet's method directly to the data containing the drift at both stations of Shionomisaki and Kyoto (II). But, concerning the principal diurnal ones, good results were obtained at Kyoto (II) alone, for there was distinct difference between the direct results and those of the data after elimination of the drift by a certain

Table 2.8. Corrected tidal factor of gravity

Method	Shionomisaki						Kyoto (II)					
	M_2	S_2	N_2	K_1	O_1	Q_1	M_2	S_2	N_2	K_1	O_1	Q_1
(d)	1.141	1.073	1.196	1.122	1.048	0.820	1.119	1.213	1.202	1.038	1.013	1.150
(e)	1.142	1.071	1.193	1.127	1.060	0.829	1.116	1.198	1.207	1.036	1.009	1.193
(bd)	1.144	1.076	1.207	1.125	1.053	0.842	1.118	1.206	1.201	1.035	1.020	1.212
(be)	1.143	1.082	1.204	1.120	1.056	0.828	1.117	1.216	1.217	1.039	1.019	1.104
(bcd)	1.140	1.079	1.199	1.123	1.078	0.811	1.113	1.200	1.213	1.036	1.017	1.136
(de)	1.140	1.079	1.202	1.121	1.077	0.805	1.114	1.204	1.206	1.037	1.014	1.150
(ade)	1.140	1.081	1.204	1.121	1.073	0.824	1.114	1.206	1.214	1.037	1.015	1.147
(bde)	1.142	1.080	1.202	1.122	1.069	0.819	1.114	1.203	1.201	1.039	1.010	1.143
(cde)	1.143	1.080	1.202	1.120	1.074	0.819	1.116	1.206	1.219	1.038	1.016	1.151
(24)	1.141	1.078	1.196	1.125	1.056	0.830	1.116	1.209	1.216	1.037	1.017	1.142
(25)	1.140	1.079	1.187	1.124	1.054	0.821	1.116	1.213	1.213	1.039	1.017	1.123

Table 2.9. Effects upon the tidal factor caused by error in the calculation process of eliminating the drift limited by observational accuracy

Shionomisaki					
M_2	S_2	N_2	K_1	O_1	Q_1
±0.003	±0.005	±0.012	±0.004	±0.006	±0.024

Kyoto (II)					
M_2	S_2	N_2	K_1	O_1	Q_1
±0.003	±0.007	±0.016	±0.003	±0.006	±0.032

method at Shionomisaki. This meant that, although the Lecolazet's method of harmonic analysis was undoubtedly excellent one, it was necessary first of all to eliminate the drift by any method prior to beginning of harmonic analysis.

Then, a real tidal factor of gravity obtained by applying the correction factor to values in Table 2.7 is shown in Table 2.8. Effects upon the tidal factor caused by error in the calculation process of eliminating the drift are shown in Table 2.9.

Tables 2.8 and 2.9 indicate that the satisfactorily coincident results can be obtained any method of drift-elimination applied. It is worth mentioning that even as simple a method as (d) can effectively available. From the above it is concluded that the same result be obtained by introducing newly the correction factor, whatever method be applied in order to eliminate the drift curve, so far as the tidal factor of gravity is concerned.

3. Harmonic analysis

Observational data of tidal phenomena consist of a composition of various constituents with proper speed. Process to obtain harmonic constants by arranging and analysing is called "harmonic analysis"

The method of harmonic analysis was established by Lord Kelvin in 1868 (**29**). Later, G. H. Darwin (**30**) had developed and completed it, expanding the tide-generating potential in 1883, and his method had been extensively used throughout the world. In America, the U.S.A. method was published in 1893 and chiefly used by the Coast and Geodetic Survey (**31**). Further, C. Börgen (**32**) devised another one in 1894 and it was principally used in Germany.

These methods were all devised as one to investigate the oceanic tides. By applying them, observational data of the oceanic tides were extensively analysed. But harmonic constants obtained by analysis differed with time even at the same station. The investigations for this diversity were brought to clarify weak points of the method of analysis and its improvement was tried eagerly. As a result, it became clear that a constituent was disturbed each by the other. In 1921, A. T. Doodson (**33**) expanded thoroughly the tide-generating potential. At last, he devised in 1928 (**34**) a method of harmonic analysis what was called "Tidal Institute method" including

Börgen's idea, in which he succeeded in simplification of Darwin's method. Although the Tidal Institute method was originally applied to the data ranging over a year, he (**35**) made it possible to apply this method to the data for one month, in 1954. In 1958, G. W. Lennon (**36**) improved the Doodson's method and published as Lennon's. In Japan, the Tidal Institute method was also improved so as to apply it to the data extending over a month by M. Miyazaki in 1956 (**37**).

R. Lecolazet (**24**, **28**, **38**) devised an excellent method named "Lecolazet's method" as one for analysing the earth tidal data observed by gravimeter and obtained good results by applying it to the data observed by a North American gravimeter at Strasbourg (**39**). B. P. Pertzev (**40**) also published a new method of harmonic analysis for bodily tides in 1957. These modern methods of harmonic analysis differed extremely from the standard one devised by G. H. Darwin (**30**), but they were essentially a logical development of Darwin's method.

Besides them, a method of harmonic analysis was devised by P. Schureman (**41**), C. T. Suthons (**26**), W. Horn (**42**) respectively and by a few others.

In the present section, effects upon the results caused by methodical difference of harmonic analysis are in detail discussed concerning the methods of Lecolazet and Doodson-Lennon. Analytical results obtained by Darwin's method are also described for the purpose of comparison.

(1) Characteristic features of method of harmonic analysis

Concerning the three different methods of Darwin, Doodson-Lennon and Lecolazet, their features are briefly compared and discussed.

a. Fundamental principle of harmonic analysis

Darwin's method is the orthodox. It is based on calculating the hourly values of every peculiar hour for the species of tides and their Fourier analysis. On the other hand, two methods of Doodson-Lennon and Lecolazet are essentially similar. The fundamental principle of their methods is to select a linear combination so as to isolate it as far as possible a constituent with particular angular velocity to be obtained. Needless to say that there is a difference between the one selected and the other in practical calculation.

b. Elimination of drift

An instrumental drift is inevitably included in the earth tidal data observed by a gravimeter. As the Darwin's and Doodson's methods are originally devised so as to analyse the oceanic tides, they do not take into account the drift-elimination. Therefore, it is necessary to eliminate the drift prior to the harmonic analysis. But, A. T. Doodson and G. W. Lennon (**25**) have devised later a method to eliminate the drift by applying a simple linear transformation to the hourly observed values, and combined it deftly with their own method of harmonic analysis. On the other hand, as the Lecolazet's method is one to analyse originally the data observed by gravimeter, it can directly be applied to the data containing the drift. In this method, the drift is automatically eliminated in the process of analysis under assumption that it is expressed by a polynomial function of the second power with time.

Generally speaking, the Doodson-Lennon's method is preferable in case of a monotonous drift, and the Lecolazet's is suitable for irregular drift.

c. Treatment of the observational data

It is desirable that an observation is operated without interruption during the whole periods concerned. But, in practice, the interruption of data cannot be avoided. When it is within several hours, a harmonic analysis can be made keeping those parts lacked in Darwin's method, but they must be entirely interpolated in Doodson-Lennon's and Lecolazet's methods.

Hourly values after elimination of the drift must show tidal variation. When a harmonic analysis is made by using them, it is necessary in Doodson-Lennon's and Lecolazet's methods that all the hourly values are changed into positive numbers by adding a constant number. On the contrary, in case of Darwin's method, the transformation of values of negative sign into those of positive sign is not necessarily required, but it is practically convenient that the hourly read values are all changed to numbers of positive sign.

d. Epoch of analysis

The period necessitated for analysis is 30 days in any method, but there can be difference in choice of epoch. In Darwin's method, the first day of analysis period is adopted as an origin time. In Doodson-Lennon's and

Lecolazet's methods, the central day is adopted as an origin time of analysis. The hourly value after elimination of the drift is generally expressed as summations of each tidal constituent, i.e. $\Sigma fR\cos(V+u-\varepsilon)$, in which f is a factor varying in period of 18.61 years, u an angle varying in that period, V an angle changing steadily at the mean speed of the constituent, and R and ε the harmonic constants of the tidal constituent. In the Darwin's method, the value of V is adopted of the first time of the analysis period and those of u and f are adopted of the central time of its period. On the contrary, in the Doodson-Lennon's and Lecolazet's methods, the values of V, u and f are adopted of the central time of analysis period.

e. Length of analysis period

The length of analysis period can be chosen corresponding to the length of the observational data, in Darwin's method. Doodson-Lennon's method is applied to the observational data extending over a month or a year. On the contrary, Lecolazet's method is only applied to the data extending over a month. Therefore, when the observational data extending over several months are harmoniously analysed by the Doodson-Lennon's or Lecolazet's method, it must be made for several epochs. Concerning the epoch of analysis in such case, R. Lecolazet (**24**) proposed to adopt it every 21 days, but A. T. Doodson and G. W. Lennon gave no designation.

f. Degree of elimination of the effects of all other constituents in analysis for a particular constituent

There are a few constituents with almost equal speed among many constituents and they affect one another. If the period of analysis is sufficiently long, they are favourably separated and distinguished as independent waves. But the complete separation of all the constituents cannot be expected from the data of one month. It is therefore necessary to eliminate the effects of all other constituents from a constituent concerned.

In Darwin's and Doodson-Lennon's methods, they are corrected for the observed values, basing on the equilibrium theory. As to the number of constituent to be corrected, the former is several species and the latter some dozen species. On the contrary, in Lecolazet's method, the correction is not made for the observed values, but is made by adding the amplitude of minor constituents, which are impossible to separate, to theoretical values.

In this case, the constituents to be corrected amount to several tens of species. It was a principal object of R. Lecolazet to amend the analytical results for the principal constituents. He assiduously is infight with this problem.

g. Amount of labour

Although Darwin's method is the orthodox, the most plentiful labour is consumed in its calculation. In the Darwin's method, it necessitates a rewriting of the hourly values on special form for each constituent, and consequently an enormous labour is consumed. When the Darwin's method is applied, it is necessary to eliminate the drift previously. It requires also exceedingly plentiful labour, as it has been mentioned. On the contrary, in Doodson-Lennon's and Lecolazet's methods, the rewriting of the hourly values is enough once only, and the succeeding calculations are carried out by putting various stencils on this table.

Calculations are made by using real value of sine and cosine of argument in Darwin's method, while they are made by using the simple integer nearest to the real value of it in Doodson-Lennon's and Lecolazet's methods. Especially, according to Lecolazet's method, the coefficients for linear combination are only 0 and ± 1, and consequently the calculations are exceedingly simple.

Concerning the calculations of theoretical values, Darwin's and Doodson-Lennon's methods require simple calculations, but exceedingly troublesome ones will require Lecolazet's method. This is of course due to a fact that the effects of so extremely small constituents are to be excluded from a wanted constituent.

On the other hand, when the hourly theoretical values are known, in the Lecolazet's method, the theoretical amplitude and phase for each constituent are also obtained by analysing them as well as the hourly observed values. This is a clever method to avoid complexity in the theoretical calculations which is a weak point of Lecolazet's method.

(2) Application to the observational data

The earth tidal data of gravity obtained for a month at each of eleven stations in Japan by means of the Askania gravimeter No. 111 during a period of about two years from July 1957 to May 1959 were used in the present investigations (**6**). The drift curve was eliminated by Pertzev's method (**17**), first of all, from all the hourly observed values, and harmonic analyses were carried out by each of Lecolazet's and Doodson-Lennon's (second approximation) methods. Calculations of drift-elimination and harmonic analysis were carried out by an electronic computing machine 'IBM-650' at the Centre International des Marées Terrestres in Bruxelles. Details concerning them were already described in the previous article (**6**) of the present study and consequently omitted in the present one. The results of harmonic analysis were already given in Table 1.10 of Part 1 (**6**) of the present study. Although there was a difference of 18 hours in analysis epoch between Lecolazet's and Doodson-Lennon's methods, the real difference was only 3 hours in values concerned in practical calculation.

In order to investigate the methodical comparison of harmonic analysis, it was also analysed by Darwin's method (**30**) after elimination of the drift. This was not applied to the whole data obtained at eleven stations, but to the data at two stations of Kyoto (II) and Shionomisaki. The reason, why these two stations were selected, was that the drift was almost linear at the

Table 2.10. Results of harmonic analysis
Elimination of drift : Pertzev's method
Harmonic analysis : Darwin's method

Station number	Observation station	Epoch (UT)
4	Shionomisaki	Jan. 19, 00h, 1958
6	Kyoto (II)	June 12, 00h, 1958

Station number	M_2		S_2		N_2	
	G	κ	G	κ	G	κ
4	1.120±0.008	$-1^\circ.77\pm0^\circ.64$	1.095±0.008	$-5^\circ.70\pm0^\circ.78$	1.188±0.083	$-11^\circ.37\pm3^\circ.86$
6	1.095±0.009	+0.22+0.73	1.145±0.008	−0.77+1.58	1.188±0.081	+ 0.45±4.81

Station number	K_1		O_1		Q_1	
	G	κ	G	κ	G	κ
4	1.172±0.111	$-22^\circ.43\pm0^\circ.69$	1.081±0.042	$+4^\circ.30\pm3^\circ.48$	0.430±0.284	$-4^\circ.47\pm15^\circ.28$
6	0.982±0.062	−13.57±1.03	1.064±0.032	+2.45±8.41	0.877±0.202	−0.87±19.39

former station and it was exceedingly irregular at the latter station. The results of analysis obtained by Darwin's method are shown in Table 2.10.

In Table 1.10 of Part 1 (**6**) and Table 2.10 are shown the results obtained by applying the respective method of analysis after elimination of the drift by Pertzev's method. Therefore, they are not real values. The real tidal factor of gravity obtained by applying the correction factor (see Table 2.5) introduced by the author is shown in Table 2.11. The values of κ shown in Table 2.11 are cited from those of Tables 1.10 and 2.10 as they are. The values shown in Table 2.11 are the definitive results of harmonic analysis.

On the basis of the Table 2.11, the difference between the results after Lecolazet's and Doodson-Lennon's methods is calculated as Table 2.12. Especially, concerning Shionomisaki and Kyoto (II), the analytical results obtained by applying the Lecolazet's method directly based on the data containing the drift (see Table 2.7) and those by Darwin's method (see Table 2.10) are obtained. Table 2.13 shows the methodical comparison of these methods.

(3) Discussion

In the present section, the relative difference in the results caused by applying the different methods of harmonic analysis is in detail discussed, but no consideration is given to the obtained values themselves.

The effect upon the results caused by the difference in method of harmonic analysis is investigated from two standpoints, one for observed tides and the other for theoretical or calculated tides. As for the former, a great many of the analytical results are methodically compared (**20**, **43**, **44**), and as for the latter, there are researches by B. P. Pertzev and others (**45**), etc.

Speaking in more detail, P. Melchior (**43**) analysed elaborately the data, which were obtained with Askania gravimeter No. 145 at Bruxelles, by each of two different methods of Lecolazet and Doodson-Lennon. According to his results, the difference in tidal factor of gravity for M_2-constituent by two methods was within 2% and that of phase lag was within 1° except special cases. N. N. Pariisky and others (**44**) investigated the comparison for three different methods of Lecolazet, Pertzev and Doodson based on the data obtained by Askania gravimeter No. 124 at Krasnaya

Table 2.11. Corrected

Harmonic analysis :

Station number	Central epoch (UT)	M_2		S_2	
		G	κ	G	κ
1	July 14, 18h, 1957	1.142	$-1^\circ.12$	1.096	$-3^\circ.91$
2	Sept. 10, 18h, 1957	1.127	−0.89	1.103	+1.97
3	Oct. 24, 18h, 1957	1.238	−2.15	1.259	−1.73
4	Feb. 3, 18h, 1958	1.140	−0.23	1.079	−4.99
5	Apr. 16, 18h, 1958	1.196	−3.24	1.150	−4.57
6	June 27, 18h, 1958	1.114	−1.24	1.204	−1.32
7	Aug. 17, 18h, 1958	1.190	+0.36	1.132	+2.23
8	Oct. 1, 18h, 1958	1.109	+1.73	1.076	−3.58
9	Nov. 15, 18h, 1958	1.179	+0.55	1.313	−3.16
10	Feb. 23, 18h, 1959	1.160	−6.44	1.093	+8.45
11	May 4, 18h, 1959	1.127	−2.70	1.198	−3.58

Harmonic analysis :

Station number	Central epoch (UT)	M_2		S_2	
		G	κ	G	κ
1	July 14, 00h, 1957	1.145	$-1^\circ.55$	1.079	$-3^\circ.33$
2	Sept. 10, 00h, 1957	1.127	−1.33	1.102	+1.60
3	Oct. 24, 00h, 1957	1.237	−2.22	1.251	−0.67
4	Feb. 3, 00h, 1958	1.131	−0.42	1.091	−4.61
5	Apr. 16, 00h, 1958	1.196	−2.94	1.166	−4.62
6	June 27, 00h, 1958	1.113	−1.08	1.174	−1.02
7	Aug. 17, 00h, 1958	1.193	+0.45	1.139	+3.80
8	Oct. 1, 00h, 1958	1.113	+0.72	1.071	−2.44
9	Nov. 15, 00h, 1958	1.174	−1.68	1.372	+0.03
10	Feb. 23, 00h, 1959	1.146	−5.38	1.106	+5.48
11	May 4, 00h, 1959	1.137	−2.92	1.185	−4.31

Harmonic analysis :

Station number	Epoch (UT)	M_2		S_2	
		G	κ	G	κ
4	Jan. 19, 00h, 1958	1.119	$-1^\circ.77$	1.095	$-5^\circ.70$
6	June 12, 00h, 1958	1.094	+0.22	1.145	−0.77

values of G and κ

Lecolazet's method

N_2		K_1		O_1		Q_1	
G	κ	G	κ	G	κ	G	κ
1.244	$-4.^\circ50$	1.065	$+0.^\circ68$	1.167	$+0.^\circ21$	1.124	$+7.^\circ58$
1.033	−1.09	0.900	+19.40	1.008	+2.62	1.165	−9.90
1.350	−3.56	1.600	−16.99	1.293	+6.35	1.805	−13.16
1.202	−9.91	1.121	−8.66	1.077	+0.46	0.805	+8.83
1.173	−3.17	1.007	+22.89	1.276	−7.41	0.871	−4.67
1.206	−1.01	1.037	−1.41	1.014	−1.28	1.150	+13.17
1.061	−6.48	1.264	+0.30	1.347	−2.31	1.206	−2.66
1.274	−14.53	1.130	+2.26	1.150	−0.88	0.969	−14.86
1.203	−3.68	1.497	−20.58	1.127	−5.13	0.889	+0.44
1.035	+25.42	1.642	−24.70	1.174	−5.95	2.845	−0.05
0.984	−4.35	1.090	+3.29	1.067	−3.92	1.643	+6.71

Doodson-Lennon's method

N_2		K_1		O_1		Q_1	
G	κ	G	κ	G	κ	G	κ
1.274	$-5.^\circ97$	1.102	$+1.^\circ75$	1.182	$+2.^\circ04$	1.362	$+2.^\circ48$
1.048	−10.93	0.879	+21.35	1.004	+3.76	1.148	−11.87
1.262	−6.16	1.594	−15.74	1.387	+5.21	1.869	−25.96
1.157	−6.42	1.128	−6.52	1.181	−1.10	0.878	+23.59
1.204	−2.41	0.934	+23.43	1.326	−3.94	1.076	+37.30
1.166	−6.87	1.074	−0.29	0.999	−2.56	1.179	+16.21
1.123	−4.79	1.225	+0.49	1.341	+0.48	1.263	−25.92
1.178	−21.46	1.091	+5.44	1.097	+0.50	0.766	−19.56
1.130	−7.59	1.514	−20.30	1.206	+4.27	0.924	+18.03
1.025	−0.76	1.597	−22.21	1.270	−6.81	2.153	−2.15
1.013	−4.18	1.070	+3.07	1.069	−8.24	1.645	+4.62

Darwin's method

N_2		K_1		O_1		Q_1	
G	κ	G	κ	G	κ	G	κ
1.190	$-11.^\circ37$	1.172	$-22.^\circ43$	1.084	$+4.^\circ30$	0.435	$-4.^\circ47$
1.190	+0.45	0.982	−13.57	1.067	+2.45	0.886	−0.87

Table 2.12. Methodical comparison of harmonic analysis

Lecolazet — Doodson-Lennon

Station	G (percent.)						κ (degree)					
number	M_2	S_2	N_2	K_1	O_1	Q_1	M_2	S_2	N_2	K_1	O_1	Q_1
1	−0.26	+1.55	−2.41	−3.47	−1.29	−21.17	+0.43	−0.58	+ 1.47	−1.07	−1.83	+ 5.10
2	0	+0.09	−1.45	+2.33	+0.40	+ 1.46	+0.44	+0.37	+ 9.84	−1.95	−1.14	+ 1.97
3	+0.08	+0.64	+6.51	+0.38	−7.27	− 3.55	+0.07	−1.06	+ 2.60	−1.25	+1.14	+12.80
4	+0.79	−1.11	+3.74	−0.62	−9.66	− 9.07	+0.19	−0.38	− 3.49	−2.14	+1.56	−14.76
5	0	−1.39	−2.64	+7.25	−3.92	−23.54	−0.30	+0.05	− 0.76	−0.54	−3.47	−41.97
6	+0.09	+2.49	+3.32	−3.57	+1.48	− 2.52	−0.16	−0.30	+ 5.86	−1.12	+1.28	− 3.04
7	−0.25	−0.62	−5.84	+3.09	+0.45	− 4.73	−0.09	−1.57	− 1.69	−0.19	−2.79	+23.26
8	−0.36	+0.46	+7.54	+3.45	+4.61	+20.95	+1.01	−1.14	+ 6.93	−3.18	−1.38	+ 4.70
9	+0.42	−4.49	+6.07	−1.14	−7.01	− 3.94	+2.23	−3.19	+ 3.91	−0.28	−9.40	−17.59
10	+1.21	−1.19	+0.97	+2.74	−8.18	+24.32	−1.06	+2.97	+26.18	−2.49	+0.86	+ 2.10
11	−0.89	+1.09	−2.95	+1.83	−0.19	− 0.12	+0.22	+0.73	− 0.17	+0.22	+4.32	+ 2.09

Table 2.13. Methodical comparison of harmonic analysis

Observation station : Shionomisaki

Method	G (percent.)						κ (degree)					
	M_2	S_2	N_2	K_1	O_1	Q_1	M_2	S_2	N_2	K_1	O_1	Q_1
Lecolazet I—Lecolazet II	−0.26	−0.19	0	+2.23	−1.95	+12.05	−0.10	−0.12	+0.35	+ 0.09	−0.84	+ 4.40
Lecolazet I—Doodson-Lennon	+0.79	−1.11	+3.74	−0.62	−9.66	− 9.07	+0.19	−0.38	−3.49	− 2.14	+1.56	−14.76
Lecolazet I—Darwin	+1.84	−1.48	+1.00	−4.55	−0.65	+45.96	+1.54	+0.71	+1.46	+13.77	−3.84	+13.30
Doodson-Lennon—Darwin	+1.06	−0.37	−2.85	−3.90	+8.21	+50.46	+1.35	+1.09	+4.95	+15.91	−5.40	+28.06

Observation station Kyoto (II)

Method	G (percent.)						κ (degree)					
	M_2	S_2	N_2	K_1	O_1	Q_1	M_2	S_2	N_2	K_1	O_1	Q_1
Lecolazet I—Lecolazet II	0	−0.08	−0.83	−0.10	−0.30	− 4.70	0	−0.03	−0.19	− 0.11	−0.19	− 1.48
Lecolazet I—Doodson-Lennon	+0.09	+2.49	+3.32	−3.57	+1.48	− 2.52	−0.16	−0.30	+5.86	− 1.12	+1.28	− 3.04
Lecolazet I—Darwin	+1.80	+4.90	+1.33	+5.30	−5.23	+22.96	−1.46	−0.55	−1.46	+12.16	−3.73	+14.04
Doodson-Lennon—Darwin	+1.71	+2.47	−2.06	+8.57	−6.81	+24.85	−1.30	−0.25	−7.32	+13.28	−5.01	+17.08

Notes Lecolazet I—Results obtained from the data after the elimination of drift by Pertzev's method.
Lecolazet II—Results obtained from the data containing the drift.
Others —Results obtained from the data after the elimination of drift by Pertzev's method.

Pakhra. The results of which were almost similar to the Melchior's.

Among observed values for the earth tides obtained by gravimeter, it usually contains not only pure tidal constituents but also the drift of gravimeter and disturbances caused by meteorological changes. As for the instrumental drift, there is a method to eliminate it independently. But, since the meteorologically disturbed waves are superposed with the primary tidal waves, there is no method to exclude them previously. The theory of harmonic analysis is founded on two bases that amplitude ratio of the observed tides is equal to that of theoretical tides and that all constituents have an equal phase lag. But if the disturbed waves by some causes are contained, these assumptions will not be correct. In order to investigate accurately the methodical comparison, it is therefore desirable to utilize the theoretical tides. Such investigations were made by B. P. Pertzev (**45**) and K. Rinner (**23**).

For instance, B. P. Pertzev and others (**45**) have made a harmonic analysis by applying each of four different methods of Doodson, Lennon, Lacolazet and Pertzev to a set of hourly values of theoretical tides extending over 30 consecutive days. They discuss deviations of the obtained results from the theoretical values. Since their investigations are by use of the theoretical tides, no difference between the obtained and theoretical values possible in both amplitude and phase for each constituent. But, according to their results, it is reported that there existed practically differences between both values whatever the method. Their conclusions are that the four different methods of harmonic analysis turn out approximately equivalent with respect to the amount of labour needed for the calculation and that Pertzev's method of analysis has a slight advantage regarding the degree of accuracy of the attained results.

As easily be understood from Table 2.11, the values of tidal factor and phase lag obtained by analysis are considerably diverse due to the method applied, even if the same data used in analysis. With respect to M_2-, S_2-, K_1- and O_1-constituents, the maximum difference between the results by Lecolazet's method and those by Doodson-Lennon's method reaches 10% in tidal factor and 10° in phase lag. Generally speaking, when the drift is irregular, there are large differences in the results obtained by both methods. As the drift curve is originally linear, the irregularity of it means existence of many disturbances by some causes. Therefore, the methodical difference in the obtained results is perhaps attributable to the efficiency in excluding the effects of all other constituents from a wanted constituent.

Concerning the M_2-constituent, the difference between Lecolazet's and Doodson-Lennon's methods is within 1.2% in tidal factor and within 2° in phase lag in the present analysis. According to the investigation by B. P. Pertzev and others (**45**), as already described, the difference is not perfectly zero even when theoretical tides are used. In the present case, therefore, the methodical difference is no longer a subject for discussion, and must be considered to be in good agreement.

With regard to Darwin's method, it was only applied to the data obtained at Shionomisaki and Kyoto (II). As could be seen from Table 2.13, the difference between Darwin's method and either Lecolazet's or Doodson-Lennon's method, was far larger than that between Lecolazet's and Doodson-Lennon's methods. As for the M_2-constituent, there were differences by 2% in tidal factor and by 2° in phase lag between Darwin's method and either of the other two methods.

Since the drift was preliminarily eliminated by Pertzev's method prior to the harmonic analysis, these three methods, which were used in the present investigations, were quite equivalent in the matter of elimination of the drift. But, when the Lecolazet's or Doodson-Lennon's method was used in harmonic analysis, the drift was doubly eliminated because the operation of drift-elimination was contained in the process of calculation itself. It was impossible to eliminate fully the drift only by applying the Lecolazet's or Doodson-Lennon's method. It was therefore very profitable to eliminate the drift firstly by Pertzev's method and to make the harmonic analysis next by either Lecolazet's or Doodson-Lennon's method, so far as the drift-elimination was concerned.

A strong agreement was found between the results obtained by applying Lecolazet's method and those by Doodson-Lennon's method at Naze in spite of a special observation station where was on an isolated island in the Pacific Ocean. As an observation made under such conditions, one month's observation carried out at Wake Island by N. F. Ness and others (**46**) by means of LaCoste-Romberg tidal gravimeter, was cited. In that case, the data observed were analysed by each of Lecolazet's and Doodson-Lennon's methods. According to their results (**47**), almost equivalent results were obtained in tidal factor by two methods, but there was a considerable difference in the phase lag.

Regarding the degree of accuracy of the attained results, both Lecolazet's and Doodson-Lennon's methods were approximately equivalent but Darwin's method had a slight inferiority. As to the amount of labour needed for the calculation, Darwin's method was the most troublesome. Only the calculation of theoretical values in Lecolazet's method was found to be the least troublesome, because of the perfect removal of the effects of all unwanted constituents.

4. Summary

The analytical treatment of the earth tidal data observed by gravimeter, equipped with automatic recording apparatus, by usage is generally divided into three processes, the first being reading of hourly values from the registrograms ; the second, elimination of the drift ; and the third, a harmonic analysis. In the present article, the effects upon the attained analytical results caused by the difference in the methods of drift-elimination and harmonic analysis were in detail discussed.

Firstly, a study concerning determination and elimination of the drift was made from the standpoint of both the accuracy of results and the labour of calculation. After a detailed study, several methods were devised newly. Among them, there were accurate and excellent ones compared with any one hitherto adopted. These methods were applied to the earth tidal data observed by Askania gravimeter No. 111 at two stations, Kyoto (II) (the drift curve was linear) and Shionomisaki (it was irregular), and the efficiency of drift-elimination was investigated. According to the present investigations, it was clearly shown that the results of harmonic analysis were affected by the method eliminating the drift rather than by the mode of variation in drift. Furthermore, there were differences even for M_2-constituent by about 2% in tidal factor and by about 5° in phase lag, according to the method of eliminating the drift. By newly introducing the correction factor for amplitude, the author succeeded in obtaining satisfactorily coincident results for tidal factor of gravity any method of drift-elimination was applied.

In the present article, the case of small curvature of drift curve only was discussed, no account being taken into the form of the drift curve.

Various methods obtained can be applied not only to the drift-elimination from gravimetric tidal data, but also to precise determination of the mean sea level, and to reducing of the data obtained by tiltmetric or extensometric observations. Since the drift curve of gravimeter is usually linear over a long period, the above-mentioned discussions are sufficient except for special cases. However, when the drift curve has a large curvature, its form must be taken into account. On the contrary, since the zero line of tiltmeter or extensometer is violently fluctuated, the second derivative of its line is found no longer negligible and consequently a correction for the form of the zero line is required, and this problem now places under investigation to be reported in the near future.

Next, what was the effect upon the results by difference of the methods of harmonic analysis was discussed citing observational data. The gravimetric tidal data obtained at eleven stations in Japan by means of the Askania gravimeter No. 111, during a period of about two years from July 1957 to May 1959, were used as examples in the present investigations. The drift curve was eliminated by Pertzev's method from all the hourly values and a harmonic analysis was carried out by each of two different methods of Lecolazet and Doodson-Lennon (second approximation) in order to investigate the methodical difference of the harmonic analysis. Especially, concerning the two stations of Shionomisaki and Kyoto (II), a harmonic analysis was carried out by each of three different methods of Darwin, Lecolazet and Doodson-Lennon, and the attained results were compared.

Concerning the degree of accuracy of the analytical results, two methods of Lecolazet and Doodson-Lennon were approximately equivalent, but Darwin's method had a slight inferiority. Concerning the removal of the effects of other constituents, Lecolazet's method was the best and Darwin's method felt short. In view of the elimination of drift, it gave a good result to apply Lecolazet's method to the harmonic analysis. Especially, when the drift curve was monotonous, a good result could be obtained even by applying the Doodson-Lennon's method. As to the amount of labour needed for calculation, Darwin's method was the most laborious one, while Lecolazet's and Doodson-Lennon's methods were approximately equivalent in effect.

Difference between the results obtained for M_2-constituent by Lecolazet's method and those by Doodson-Lennon's method was within 1% in tidal factor and 2° in phase lag. As for the other constituents, certainly it was considerably larger. Generally speaking, the methodical difference between Lecolazet's and Doodson-Lennon's was large at a place where the change of meteorological elements was violent. Under these circumstances, the attained results differed considerably, in unfavourable case, by the difference of method of harmonic analysis even when the same data were used. This fact means that a careful consideration is necessitated in case of discussing in detail an internal constitution of the earth based on analytical results.

From these considerations, it is concluded that it is the most desirable to use the Lecolazet's method in harmonic analysis among the three methods of Darwin, Lecolazet and Doodson-Lennon. Since the form of drift curve changes momentarily, it is impossible to eliminate fully the drift by applying any method only once to the observational data, though what kind of method of drift-elimination is applied. In Lecolazet's method, the drift is automatically eliminated in the process of analysis, but it is also impossible to eliminate fully the drift by this method only. The following therefore can be mentioned as conclusions of the present investigations that it is recommendable in analytical treatments of the observational data obtained by gravimeter :

(1) To determine and eliminate the drift by any method before commencing the harmonic analysis.

(2) To analyse harmoniously the data by Lecolazet's method.

(3) To correct the amplitude (or tidal factor of gravity) obtained by the analysis, by applying the correction factor introduced by the author.

In order to compare the results obtained at many stations in the world one another, it is necessary to treat the observational data by the same method. Centre International des Marées Terrestres analyses them by the same method using an electronic computing machine. The contribution of the Centre of this realm must be highly appreciated.

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